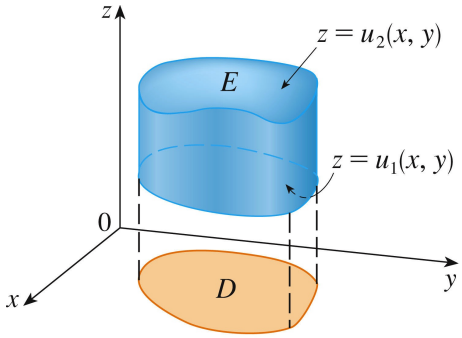
Lesson 27c. Triple Integrals, cont.

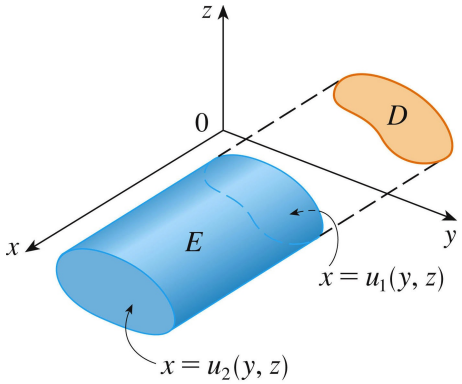
1 Last time: integrating over general 3D regions

- **Type A 3D region**



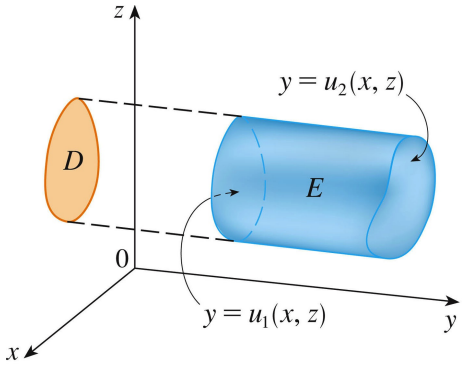
$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz \right] dA$$

- **Type B 3D region**



$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x,y,z)\,dx \right] dA$$

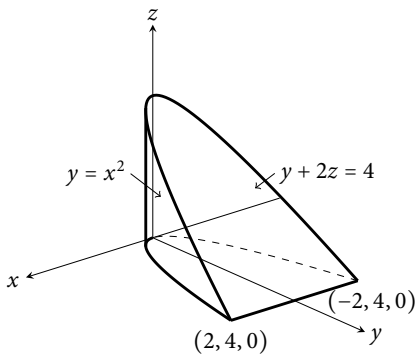
- **Type C 3D region**

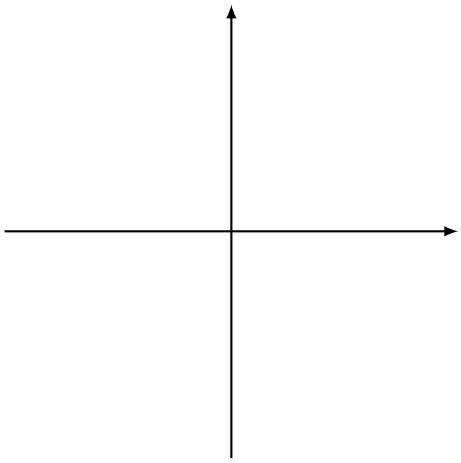


$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x,y,z)\,dy \right] dA$$

2 Using different orders of integration

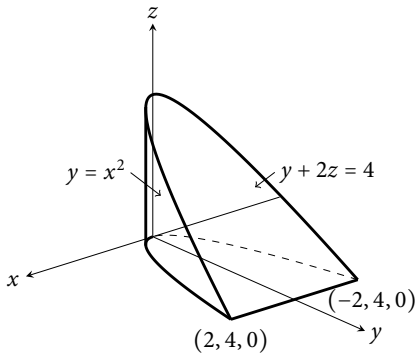
Example 1. Draw the projection of the 3D region below onto the xy-plane.

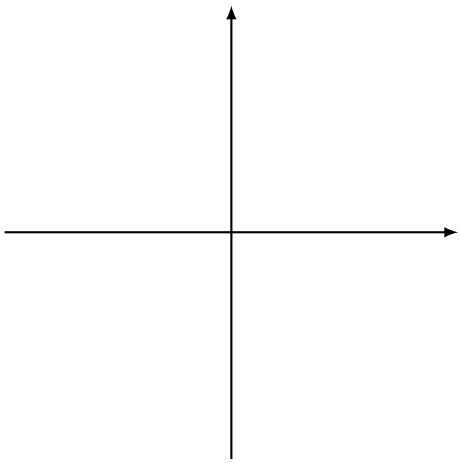




Example 2. Express $\iiint_E f(x, y, z)\, dV$ as an iterated integral, where E is the region in Example 1, using the order of integration $dz\, dx\, dy$.

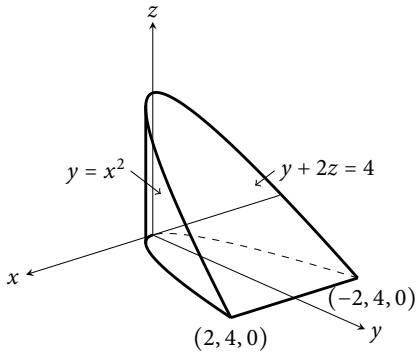
Example 3. Draw the projection of the 3D region in Example 1 (below) onto the yz-plane.

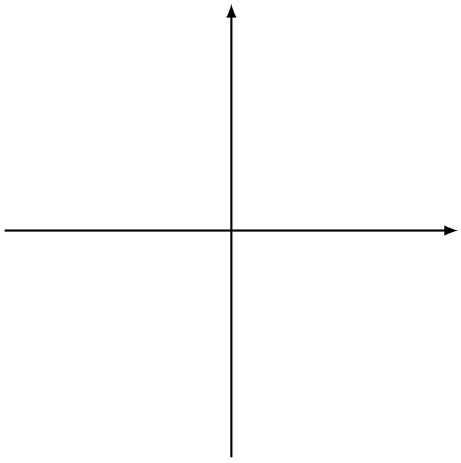




Example 4. Express $\iiint_E f(x, y, z)\, dV$ as an iterated integral, where E is the region in Example 1, using the order of integration $dx\, dz\, dy$.

Example 5. Draw the projection of the 3D region in Example 1 (below) onto the xz-plane.





Example 6. Express $\iiint_E f(x, y, z)\, dV$ as an iterated integral, where E is the region in Example 1, using the order of integration $dy\, dz\, dx$.

3 If we have time...

Example 7. Express $\iiint_E \sqrt{x^2 + z^2}\, dV$ as an iterated integral, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.